

Addition

Year 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign.

Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$2 = 1 + 1$$

$$2 + 3 = 4 + 1$$

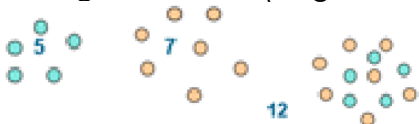
Missing numbers need to be placed in all possible places.

$$3 + 4 = \square \quad \square = 3 + 4$$

$$3 + \square = 7 \quad 7 = \square + 4$$

Counting and Combining sets of Objects

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)

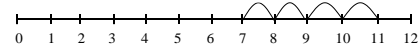


Understanding of counting on with a numbertrack.



Understanding of counting on with a numberline.

(supported by cups, images and other concrete materials).



$$7 + 4$$

Year 2

Missing number problems e.g $14 + 5 = 10 + \square$
 $32 + \square + \square = 100$ $35 = 1 + \square + 5$

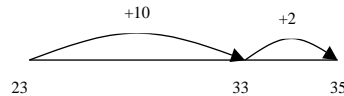
It is valuable to use a range of representations (also see Y1). Continue to use numberlines to develop understanding of:

Counting on in tens and ones

$$23 + 12 = 23 + 10 + 2$$

$$= 33 + 2$$

$$= 35$$

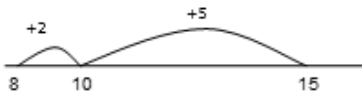


Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10

e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

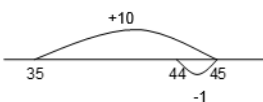
$$8 + 7 = 15$$



Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

$$35 + 9 = 44$$



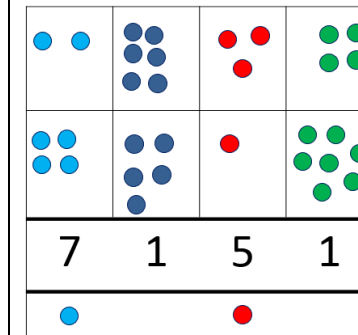
Year 3

Compact written method

Extend to numbers with at least four digits.

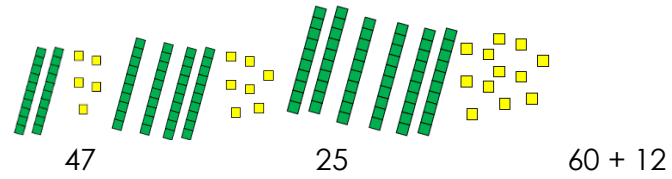
Extend to up to two places of decimals (same number of decimal places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

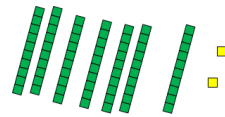


$$\begin{array}{r} 2634 \\ + 4517 \\ \hline 7151 \\ \hline 1 \quad 1 \end{array}$$

Towards a Written Method
Partitioning in different ways and recombine
 $47+25$



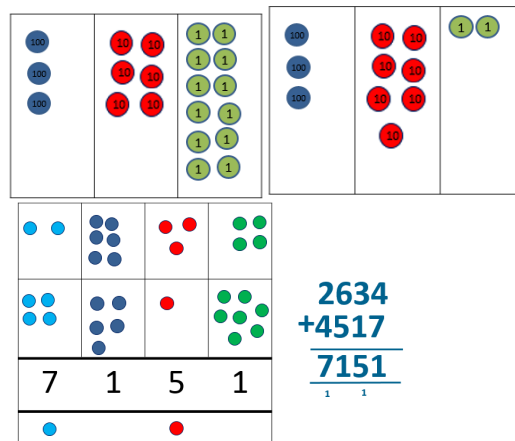
Leading to exchanging:



72

Towards a Written Method

Introduce column addition modelled with concrete objects (Dienes could be used for those who need a less abstract representation) Leading to children understanding the exchange between tens and ones/tens and hundreds.
 Children begin to use a formal columnar algorithm.



Year 1

Year 2

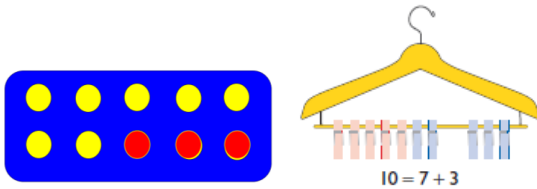
Year 3

Mental Strategies (addition and subtraction)

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

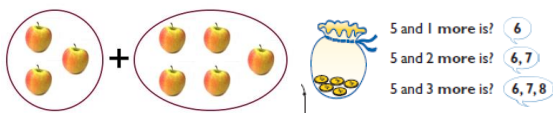
They should see addition and subtraction as related operations. E.g. $7 + 3 = 10$ is related to $10 - 3 = 7$, understanding of which could be supported by an image like this.



Use bundles of straws, cups and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value.

Children have opportunities to explore partitioning numbers in different ways. e.g. $7 = 6 + 1$, $7 = 5 + 2$, $7 = 4 + 3 =$

Children should begin to understand addition as combining groups and counting on.



Vocabulary

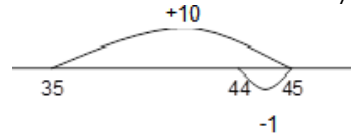
Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on. 'Get ready to get some more'

Generalisations

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10.

Number lines should be used in conjunction with cups to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting.



Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g. using $7 + 3 = 10$ to find $17 + 3 = 20$, $70 + 30 = 100$

They should use concrete objects such as cups, bead strings and number lines to explore missing numbers $- 45 + = 50$.

As well as cups and number lines, 100 squares could be used to explore patterns in calculations such as $74 + 11$, $77 + 9$ encouraging children to think about 'What do you notice?' where partitioning or adjusting is used.

Children should learn to check their calculations, by using the inverse.

They should continue to see addition as both combining groups and counting on.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. $23 = 20 + 3 = 10 + 13$.

Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, Tens, ones, partition Near multiple of 10, tens boundary, More than, one more, two more... ten more... one hundred more

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.

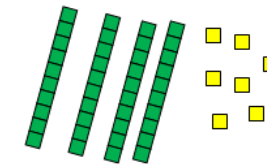
Children should continue to partition numbers in different ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.

Add the nearest multiple of 10, then adjust such as $63 + 29$ is the same as $63 + 30 - 1$; counting on by partitioning the second number only such as $72 + 31 = 72 + 30 + 1 = 102 + 1 = 103$

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg.

What's the same? What's different?



Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

(Links between addition and subtraction)
When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Another example here...

Some Key Questions

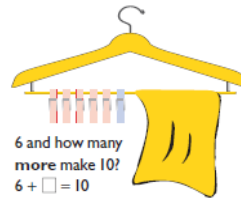
How many altogether? How many more to make...? I add ...more. What is the total? How many more is... than...? How much more is...? One more, two more, ten more...
What can you see here?
Is this true or false?
What is the same? What is different?

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$7 + ? = 10$$



Some Key Questions

How many altogether? How many more to make...?
How many more is... than...? How much more is...?
Is this true or false?
If I know that $17 + 2 = 19$, what else do I know? (e.g. $2 + 17 = 19$; $19 - 17 = 2$; $19 - 2 = 17$; $190 - 20 = 170$ etc).
What do you notice? What patterns can you see?

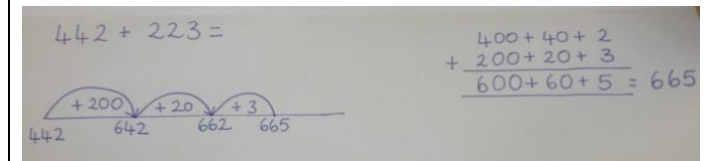
Generalisations

Noticing what happens to the digits when you count in tens and hundreds.
Odd + odd = even etc (see Year 2)
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?



Year 4	Year 5	Year 6
<u>Written methods (progressing to more than 4-digits)</u>	<u>Mental methods</u> should continue to develop, supported by a range of models and images,	<u>Mental methods</u> should continue to develop, supported by a range of models and images,

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

$$\begin{array}{r} 172.83 \\ + \underline{54.68} \\ \hline 227.51 \\ \text{11 1} \end{array}$$

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency
e.g. $12462 + 2300 = 14762$

Year 4

including the number line.

Written methods

Children will progress to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.

Continue calculating with decimals, including those with different numbers of decimal places

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

Year 5

including the number line.

Written methods

As with Year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.

Continue calculating with decimals, including those with different numbers of decimal places

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

Year 6

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: $124 - 47$, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: $28 + 75$, $75 + 28$ (thinking of 28 as $25 + 3$)
- Partitioning: counting on or back: $5.6 + 3.7$, $5.6 + 3 + 0.7 = 8.6 + 0.7$
- Partitioning: bridging through multiples of 10: $6070 - 4987$, $4987 + 13 + 1000 + 70$
- Partitioning: compensating - $138 + 69$, $138 + 70 - 1$
- Partitioning: using 'near' doubles - $160 + 170$ is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval - What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: $1.7 + 0.55$
- Reordering: $4.7 + 5.6 - 0.7$, $4.7 - 0.7 + 5.6 = 4 + 5.6$
- Partitioning: counting on or back - $540 + 280$, $540 + 200 + 80$
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: $5.7 + 3.9$, $5.7 + 4.0 - 0.1$
- Partitioning: using 'near' double: $2.5 + 2.6$ is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

Vocabulary

tens of thousands boundary,
Also see previous years

Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.

What do you notice about the differences between consecutive square numbers?

[Investigate \$a - b = \(a-1\) - \(b-1\)\$ represented visually.](#)

Some Key Questions

Mental Strategies

Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$

Vocabulary

See previous years

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Sometimes, always or never true? Subtracting numbers makes them smaller.

Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

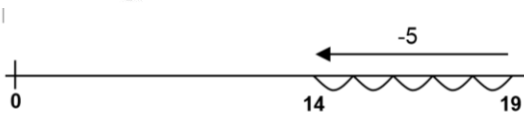
<p>Generalisations Investigate when re-ordering works as a strategy for subtraction. Eg. $20 - 3 - 10 = 20 - 10 - 3$, but $3 - 20 - 10$ would give a different answer.</p> <p>Some Key Questions What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p>What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	
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Subtraction		
Year 1	Year 2	Year 3

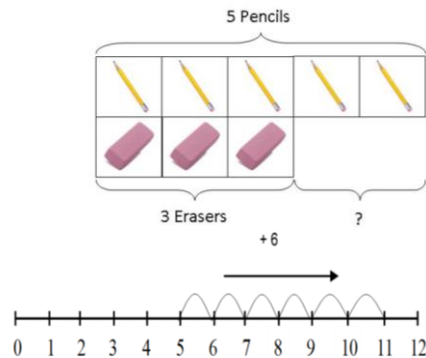
Missing number problems e.g. $7 = \square - 9$; $20 - \square = 9$; $15 - 9 = \square$; $\square - \square = 11$; $16 - 0 = \square$

Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

Understand subtraction as take-away:



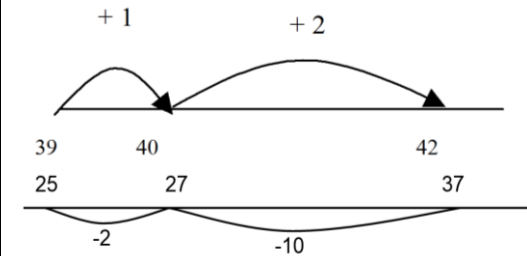
Understand subtraction as finding the difference:



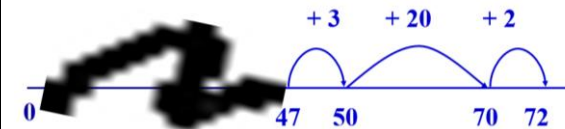
The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation. The use of other images is also valuable for modelling subtraction e.g. Numicon, cups bundles of straws, Dienes apparatus, multi-link cubes, bead strings

Year 1

Missing number problems e.g. $52 - 8 = \square$; $\square - 20 = 25$; $22 = \square - 21$; $6 + \square + 3 = 11$

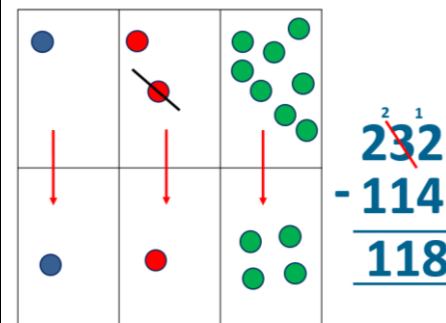


It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference. E.g. The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



Written methods (progressing to 4-digits)

Expanded column subtraction with decomposition, progressing to calculations with 4-digit numbers.



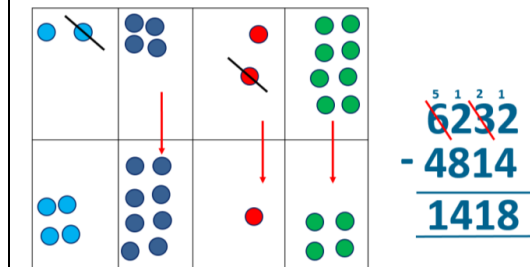
Year 2

Missing number/digit problems: $456 + \square = 710$; $1\square7 + 6\square = 200$; $60 + 99 + \square = 340$; $200 - 90 - 80 = \square$; $225 - \square = 150$; $\square - 25 = 67$; $3450 - 1000 = \square$; $\square - 2000 = 900$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.



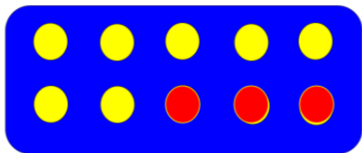
Year 3

Mental Strategies

Children should experience [regular counting](#) on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g. $7 + 3 = 10$ is related to $10 - 3 = 7$, understanding of which could be supported by an image like this.

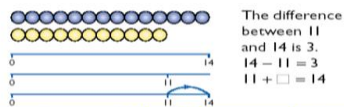


Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.



Subtraction as "taking away"



Subtraction as "the difference between"

Vocabulary

Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

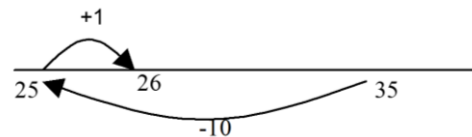
Generalisations

- True or false? Subtraction makes

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.



Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $100 - 70 = 30$ and $70 = 100 - 30$.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

As well as number lines, 100 squares could be used to model calculations such as $74 - 11$, $77 - 9$ or $36 - 14$, where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that 'up' equates to 'add'.

Children should learn to check their calculations, including by adding to check.

They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. $23 = 20 + 3 = 10 + 13$.

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Children should continue to partition numbers in difference ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for $201 - 198$; counting back (taking away / partition into tens and ones) for $201 - 12$.

Calculators can usefully be introduced to encourage fluency by using them for games such as 'Zap' [e.g. Enter the number 567. Can you 'zap' the 6 digit and make the display say 507 by subtracting 1 number?]

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd - odd = even etc (see Year 2)

Inverses and related facts - develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

numbers smaller

- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, "What can you see here?" e.g.

3 yellow, 1 red, 1 blue. $3 + 1 + 1 = 5$

2 circles, 2 triangles,

1 square. $2 + 2 + 1 =$

5

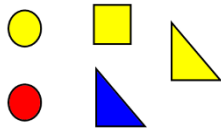
I see 2 shapes with

curved lines and 3

with straight lines. $5 =$

$2 + 3$

$5 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 3$



Some Key Questions

How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many have gone? One less, two less, ten less... How many fewer is... than...? How much less is...?

What can you see here?

Is this true or false?

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus

Tens, ones, partition

Near multiple of 10, tens boundary

Less than, one less, two less... ten less... one hundred less

More, one more, two more... ten more... one

hundred more **Generalisation**

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd - odd = even; odd - even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$15 + 5 = 20$$

Some Key Questions

How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many fewer is... than...? How much less is...?

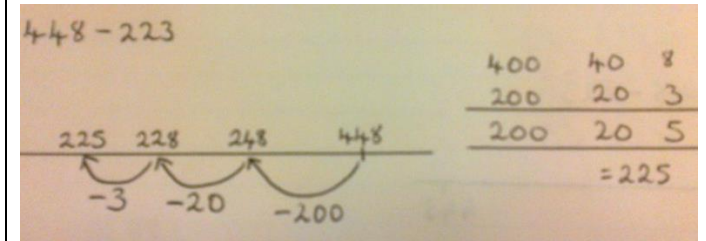
Is this true or false?

If I know that $7 + 2 = 9$, what else do I know? (e.g. $2 + 7 = 9$; $9 - 7 = 2$; $9 - 2 = 7$; $90 - 20 = 70$ etc).

What do you notice? What patterns can you see?

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line



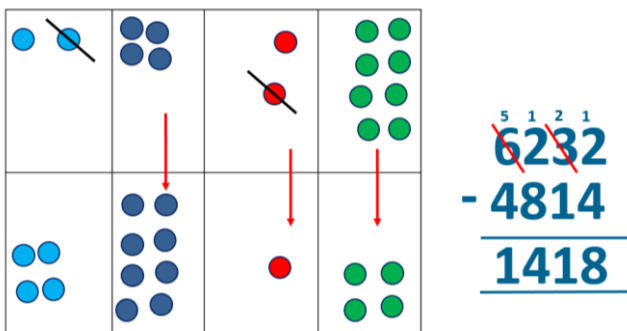
Year 4	Year 5	Year 6
Missing number/digit problems: $6.45 = 6 + 0.4$	Missing number/digit problems: □ and # each	Missing number/digit problems: □ and # each

$+ \square$; $119 - \square = 86$; $1\,000\,000 - \square = 999\,000$; $600\,000 + \square + 1000 = 671\,000$; $12\,462 - 2\,300 = \square$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to more than 4-digits)

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.



Year 4

stand for a different number. $\# = 34$. $\# + \# = \square + \square + \#$. What is the value of \square ?

$10\,000\,000 = 9\,000\,100 + \square$
 $7 - 2 \times 3 = \square$; $(7 - 2) \times 3 = \square$; $(\square - 2) \times 3 = 15$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.

Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example:

$$\begin{array}{r} 326 \\ -148 \\ -2 \\ -20 \\ \hline 200 \\ \hline 178 \end{array}$$

Year 5

stand for a different number. $\# = 34$. $\# + \# = \square + \square + \#$. What is the value of \square ?

$10\,000\,000 = 9\,000\,100 + \square$
 $7 - 2 \times 3 = \square$; $(7 - 2) \times 3 = \square$; $(\square - 2) \times 3 = 15$

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.

Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example:

$$\begin{array}{r} 326 \\ -148 \\ -2 \\ -20 \\ \hline 200 \\ \hline 178 \end{array}$$

Year 6

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: $124 - 47$, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: $28 + 75$, $75 + 28$ (thinking of 28 as $25 + 3$)
- Partitioning: counting on or back: $5.6 + 3.7$, $5.6 + 3 + 0.7 = 8.6 + 0.7$
- Partitioning: bridging through multiples of 10: $6070 - 4987$, $4987 + 13 + 1000 + 70$
- Partitioning: compensating – $138 + 69$, $138 + 70 - 1$
- Partitioning: using 'near' doubles - $160 + 170$ is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: $1.7 + 0.55$
- Reordering: $4.7 + 5.6 - 0.7$, $4.7 - 0.7 + 5.6 = 4 + 5.6$
- Partitioning: counting on or back - $540 + 280$, $540 + 200 + 80$
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: $5.7 + 3.9$, $5.7 + 4.0 - 0.1$
- Partitioning: using 'near' double: $2.5 + 2.6$ is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

Vocabulary

tens of thousands boundary,
Also see previous years

Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.

What do you notice about the differences between consecutive square numbers?

[Investigate \$a - b = \(a-1\) - \(b-1\)\$ represented visually.](#)

Some Key Questions

Mental Strategies

Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$

Vocabulary

See previous years

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Sometimes, always or never true? Subtracting numbers makes them smaller.

Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

Generalisations

Investigate when re-ordering works as a strategy for subtraction. Eg. $20 - 3 - 10 = 20 - 10 - 3$, but $3 - 20 - 10$ would give a different answer.

Some Key Questions

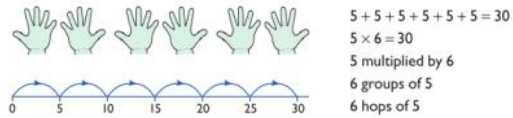
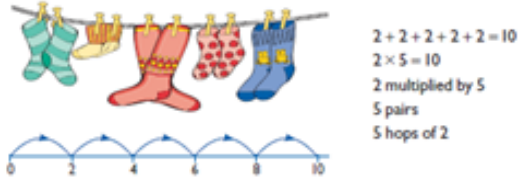
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Multiplication		
Year 1	Year 2	Year 3

Understand multiplication is related to doubling and combining groups of the same size (repeated addition)

Washing line, and other practical resources for counting. Concrete objects. Numicon, cups, bundles of straws, bead strings

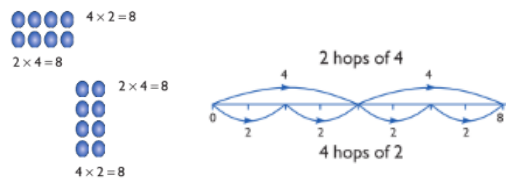


Problem solving with concrete objects

(including money and measures)
 Use cuisenaire and bar method to develop the vocabulary relating to 'times' –

Pick up five, 4 times

Use arrays to understand multiplication can be done in any order (commutative)

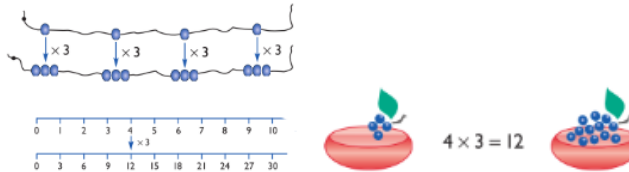


Expressing multiplication as a number sentence using x

Using understanding of the inverse and practical resources to solve missing number problems.

$7 \times 2 = \square$ $\square = 2 \times 7$
 $7 \times \square = 14$ $14 = \square \times 7$
 $\square \times 2 = 14$ $14 = 2 \times \square$
 $\square \times \bigcirc = 14$ $14 = \square \times \bigcirc$

Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables. Begin to develop understanding of multiplication as scaling (3 times bigger/taller)



Doubling numbers up to 10 + 10

Link with understanding scaling. Using known doubles to work out double 2d numbers (double 15 = double 10 + double 5)

Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images

Missing number problems

Continue with a range of equations as in Year 2 but with appropriate numbers.

Mental methods

Doubling 2 digit numbers using partitioning
 Demonstrating multiplication on a number line – jumping in larger groups of amounts
 $13 \times 4 = 10$ groups 4 = 3 groups of 4

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

$\square \times 5 = 160$

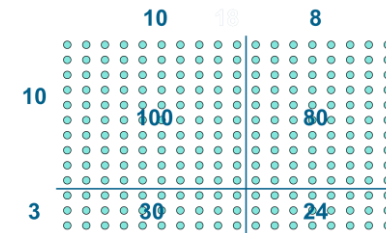
Mental methods

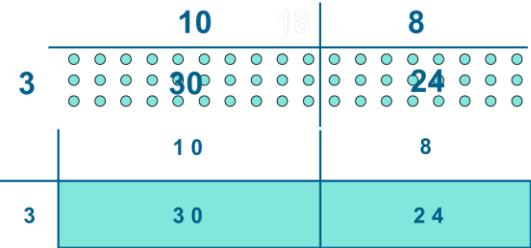
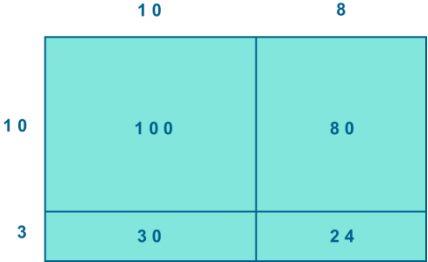

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

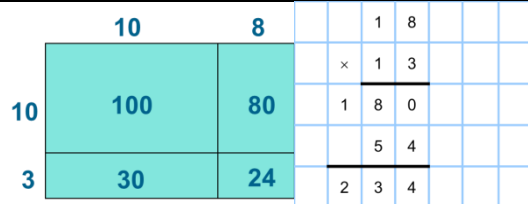
Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.



	<p>Develop onto the grid method</p>  <p>Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters</p>	
Year 1	Year 2	Year 3
<p><u>Mental Strategies</u> Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should memorise and reason with numbers in 2, 5 and 10 times tables They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.</p>  <p>Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower)</p> <p><u>Vocabulary</u> Ones, groups, lots of, doubling repeated addition groups of, lots of, times, columns, rows longer, bigger, higher etc times as (big, long, wide ...etc)</p>	<p><u>Mental Strategies</u> Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Number lines should continue to be an important image to support thinking, for example</p> <p>Children should practise times table facts $2 \times 1 =$ $2 \times 2 =$ $2 \times 3 =$</p> <p>Use a clock face to support understanding of counting in 5s. Use money to support counting in 2s, 5s, 10s, 20s, 50s</p> <p><u>Vocabulary</u> multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows</p> <p><u>Generalisation</u> Commutative law shown on array (video) Repeated addition can be shown mentally on a number line</p> <p>Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.</p>	<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.</p> <p>Children should practise times table facts $3 \times 1 =$ $3 \times 2 =$ $3 \times 3 =$</p> <p><u>Vocabulary</u> partition grid method inverse</p> <p><u>Generalisations</u> Connecting $\times 2$, $\times 4$ and $\times 8$ through multiplication facts</p> <p>Comparing times tables with the same times tables which is ten times bigger. If $4 \times 3 = 12$, then we know $4 \times 30 = 120$. Use place value counters to demonstrate this.</p>

<p>Generalisations Understand 6 counters can be arranged as 3+3 or 2+2+2</p> <p>Understand that when counting in twos, the numbers are always even.</p> <p>Some Key Questions Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p>Some Key Questions What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p>When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)</p> <p>Some Key Questions What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>																														
Year 4	Year 5	Year 6																														
<p>Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits</p> <p>Mental methods X by 10, 100, 1000 using moving digits Use practical resources and jottings to explore equivalent statements (e.g. $4 \times 35 = 2 \times 2 \times 35$)</p> <p>Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning) Solving practical problems where children need to scale up. Relate to known number facts. Identify factor pairs for numbers</p> <p>Written methods (progressing to 4d x 2d) Long multiplication using place value counters Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)</p>	<p>Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits</p> <p>Mental methods Identifying common factors and multiples of given numbers Solving practical problems where children need to scale up. Relate to known number facts.</p> <p>Written methods Continue to refine and deepen understanding of written methods including fluency for using long multiplication</p> <table border="1" data-bbox="763 1114 1232 1334"> <tbody> <tr> <td>X</td> <td>1000</td> <td>300</td> <td>40</td> <td>2</td> </tr> <tr> <td>10</td> <td>10000</td> <td>3000</td> <td>400</td> <td>20</td> </tr> <tr> <td>8</td> <td>8000</td> <td>2400</td> <td>320</td> <td>16</td> </tr> </tbody> </table>	X	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16	<p>Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits</p> <p>Mental methods Identifying common factors and multiples of given numbers Solving practical problems where children need to scale up. Relate to known number facts.</p> <p>Written methods Continue to refine and deepen understanding of written methods including fluency for using long multiplication</p> <table border="1" data-bbox="1568 1150 2038 1370"> <tbody> <tr> <td>X</td> <td>1000</td> <td>300</td> <td>40</td> <td>2</td> </tr> <tr> <td>10</td> <td>10000</td> <td>3000</td> <td>400</td> <td>20</td> </tr> <tr> <td>8</td> <td>8000</td> <td>2400</td> <td>320</td> <td>16</td> </tr> </tbody> </table>	X	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
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$$\begin{array}{r}
 ^2 ^3 ^1 \\
 1342 \\
 \times 18 \\
 \hline
 13420 \\
 10736 \\
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 24156 \\
 ^1
 \end{array}$$

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 ^2 ^3 ^1 \\
 1342 \\
 \times 18 \\
 \hline
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 \hline
 24156 \\
 ^1
 \end{array}$$

Year 4

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
 Become fluent and confident to recall all tables to $\times 12$
 Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?)
 Use of finger strategy for 9 times table.

Multiply 3 numbers together
 The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies:

- Partitioning using $\times 10$, $\times 20$ etc
- Doubling to solve $\times 2$, $\times 4$, $\times 8$
- Recall of times tables
- Use of commutativity of multiplication

Vocabulary

Factor

Generalisations

Children given the opportunity to investigate numbers multiplied by 1 and 0.

Year 5

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.
 Multiply by 10, 100, 1000, including decimals (Moving Digits ITP)
 The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using $\times 10$, $\times 20$ etc
- Doubling to solve $\times 2$, $\times 4$, $\times 8$
- Recall of times tables
- Use of commutativity of multiplication

If children know the times table facts to 12×12 . Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)

Vocabulary

cube numbers
 prime numbers
 square numbers
 common factors
 prime number, prime factors
 composite numbers

Generalisation

Year 6

Mental Strategies

Consolidate previous years.
 Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$
 They should be encouraged to choose from a range of strategies to solve problems mentally:

- Partitioning using $\times 10$, $\times 20$ etc
- Doubling to solve $\times 2$, $\times 4$, $\times 8$
- Recall of times tables
- Use of commutativity of multiplication

If children know the times table facts to 12×12 . Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)

Vocabulary

See previous years
 common factor

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acronym such as PEMDAS, or could be encouraged to design their own ways of

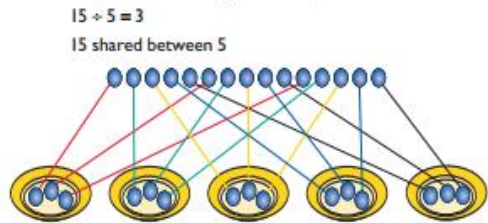
<p>When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p>Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?</p>	<p>remembering. Understanding the use of multiplication to support conversions between units of measurement.</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>
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Division		
Year 1	Year 2	Year 3

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.

Children should be given opportunities to reason about what they notice in number patterns.

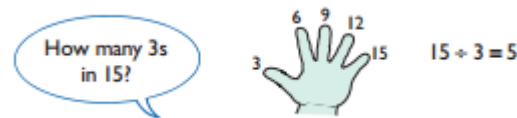
Group AND share small quantities- understanding the difference between the two concepts.



Sharing

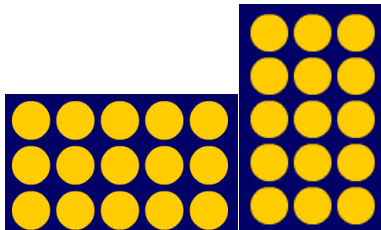
Develops importance of one-to-one correspondence.

Children should be taught to share using concrete apparatus.



Grouping

Children should apply their counting skills to develop some understanding of grouping. Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3.



15 ÷ 5 = 3 There are 3 groups of 5.

Children should be able to find 1/2 and 1/4 and simple fractions of objects, numbers

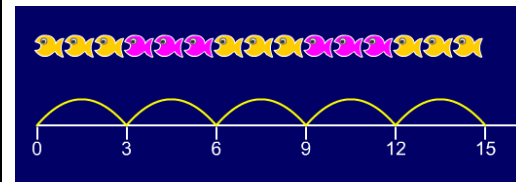
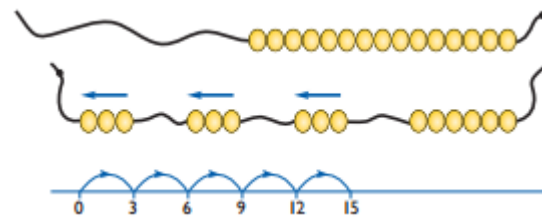
÷ = signs and missing numbers

6 ÷ 2 = □ □ = 6 ÷ 2
6 ÷ □ = 3 3 = 6 ÷ □
□ ÷ 2 = 3 3 = □ ÷ 2
□ ÷ ▽ = 3 3 = □ ÷ ▽

Know and understand sharing and grouping- introducing children to the ÷ sign. Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a numberline

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.
15 ÷ 3 = 5



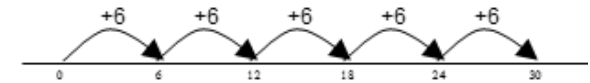
Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

÷ = signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

Grouping

How many 6's are in 30?
30 ÷ 6 can be modelled as:



Becoming more efficient using a numberline

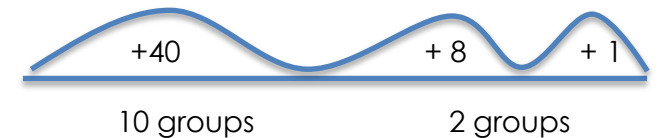
Children need to be able to partition the dividend in different ways.

48 ÷ 4 = 12



Remainders

49 ÷ 4 = 12 r1



Sharing – 49 shared between 4. How many left over?

Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.

For example:

60 ÷ 10 = How many groups of 10 in 60?

600 ÷ 100 = How many groups of 100 in 600?

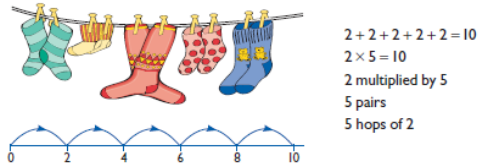
and quantities.

Year 1

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



Children should begin to understand division as both sharing and grouping.

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Grouping-
How many 2's are in 6?



They should use objects to group and share amounts to develop understanding of division in a practical sense.

E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves?

Children should begin to explore finding simple fractions of objects, numbers and quantities.

E.g. 16 children went to the park at the weekend. Half that number went swimming. How many

Year 2

Mental Strategies

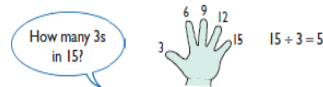
Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2×6 , 5×4 , 10×9 . Show the children how to hold out their fingers and count, touching each finger in turn. So for 2×6 (six twos), hold up 6 fingers:



Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.

Children should continue to develop understanding of division as sharing **and** grouping.



15 pencils shared between 3 pots, how many in each pot?

Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.

They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

Year 3

Mental Strategies

Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables. This then helps them to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables).

Children will make use multiplication and division facts they know to make links with other facts.
 $3 \times 2 = 6$, $6 \div 3 = 2$, $2 = 6 \div 3$
 $30 \times 2 = 60$, $60 \div 3 = 20$, $2 = 60 \div 30$

They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to be given as a whole number) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

Use children's intuition to support understanding of fractions as an answer to a sharing problem.

3 apples shared between 4 people = $\frac{3}{4}$



Vocabulary

See Y1 and Y2
inverse

Generalisations

Inverses and related facts – develop fluency in finding related multiplication and division facts. Develop the knowledge that the inverse relationship can be used as a checking method.

children went swimming?

Vocabulary

share, share equally, one each, two each..., group, groups of, lots of, array

Generalisations

- True or false? I can only halve even numbers.
- Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing.

Some Key Questions

How many groups of...?
How many in each group?
Share... equally into...
What can do you notice?

Use children's intuition to support understanding of fractions as an answer to a sharing problem.

3 apples shared between 4 people = $\frac{3}{4}$



Vocabulary

group in pairs, 3s ... 10s etc
equal groups of
divide, ÷, divided by, divided into, remainder

Generalisations

Noticing how counting in multiples of 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)

An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)

Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.

Some Key Questions

How many 10s can you subtract from 60?
I think of a number and double it. My answer is 8. What was my number?
If $12 \times 2 = 24$, what is $24 \div 2$?
Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)

Some Key Questions

Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)
What is the missing number? $17 = 5 \times 3 + \underline{\quad}$
 $\underline{\quad} = 2 \times 8 + 1$

Year 4

÷ = signs and missing numbers

Continue using a range of equations as in year 3 but with appropriate numbers.

Year 5

÷ = signs and missing numbers

Continue using a range of equations but with appropriate numbers

Year 6

÷ = signs and missing numbers

Continue using a range of equations but with appropriate numbers

Sharing and Grouping

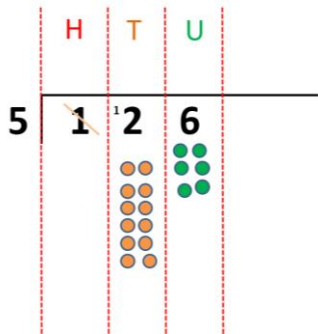
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:

Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1



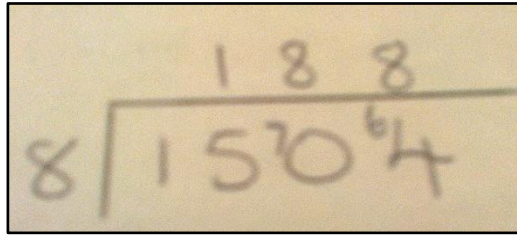
Formal Written Methods

Sharing and Grouping

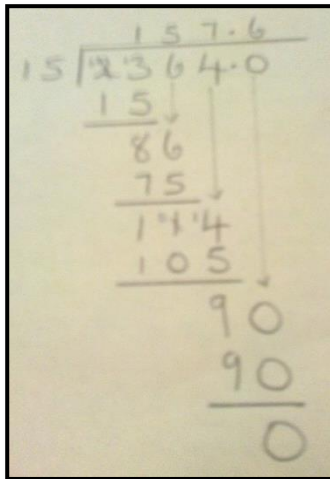
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate. Quotients should be expressed as decimals and fractions

Formal Written Methods – long and short division

E.g. $1504 \div 8$



E.g. $2364 \div 15$



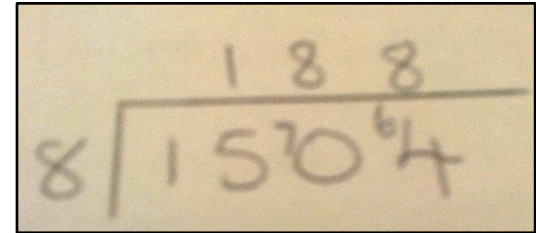
Formal Written Methods

Sharing and Grouping

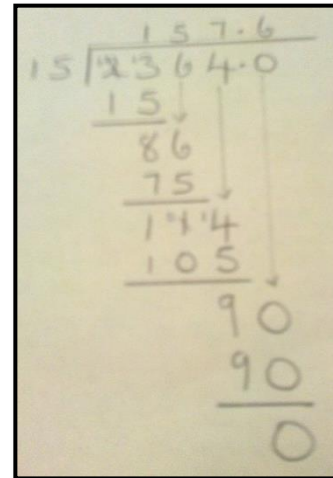
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate. Quotients should be expressed as decimals and fractions

Formal Written Methods – long and short division

E.g. $1504 \div 8$



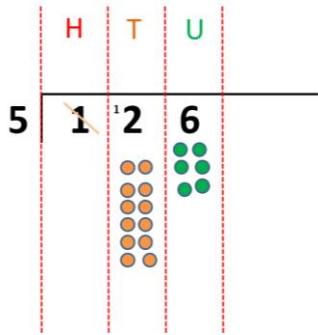
E.g. $2364 \div 15$



Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

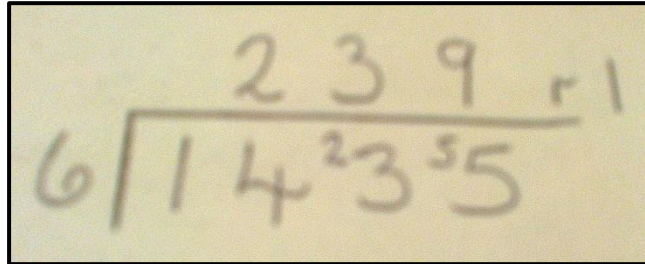
Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1



Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4)

E.g. $1435 \div 6$

Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1?)



How could I share this between 6 as well?

Year 4

Mental Strategies

Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000.
Children should learn the multiplication facts to 12 x 12.

Vocabulary

see years 1-3
divide, divided by, divisible by, divided into
share between, groups of
factor, factor pair, multiple
times as (big, long, wide ...etc)
equals, remainder, quotient, divisor
inverse

Towards a formal written method

Year 5

Mental Strategies

Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency.
Children should practice and apply the multiplication facts to 12 x 12.

Vocabulary

see year 4
common factors
prime number, prime factors
composite numbers
short division
square number
cube number
inverse
power of

Year 6

Mental Strategies

Children should count regularly, building on previous work in previous years.
Children should practice and apply the multiplication facts to 12 x 12.

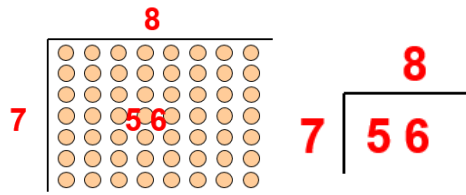
Vocabulary

see years 4 and 5

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.



Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.

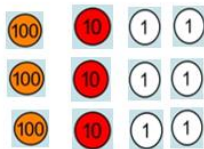
Each digit as a multiple of the divisor

'How many groups of 3 are there in the hundreds column?'

'How many groups of 3 are there in the tens column?'

'How many groups of 3 are there in the units/ones column?'

$$\begin{array}{r} 112 \\ 3 \overline{) 336} \\ \underline{33} \\ 6 \\ \underline{6} \\ 0 \end{array}$$



When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto 'carrying' their remainder across to the next digit.

Generalisations

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Generalisations

The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.

Start: $24 = 24$

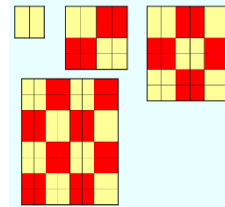
Player 1: $4 \times 6 = 24$

Player 2: $4 \times 6 = 12 \times 2$

Player 1: $48 \div 2 = 12 \times 2$

Sometimes, always, never true questions about multiples and divisibility. E.g.:

- If the last two digits of a number are divisible by 4, the number will be divisible by 4.
- If the digital root of a number is 9, the number will be divisible by 9.
- When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)



Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)

Using what you know about [rules of divisibility](#), do you think 7919 is a prime number? Explain your answer.

Some Key Questions for Year 4 to 6
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Is it sometimes, always or never true that $\square \div \Delta = \Delta \div \square$?

Inverses and deriving facts. 'Know one, get lots free!' e.g.: $2 \times 3 = 6$, so $3 \times 2 = 6$, $6 \div 2 = 3$, $60 \div 20 = 3$, $600 \div 3 = 200$ etc.

Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.